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Numerical Pattern Questions: Analytical Strategies And Teaching Implications

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***Abstract**:*
Numerical law questions are open-ended, which is an effective carrier for developing students' logical reasoning ability and innovative thinking. The strategies for coping with the numerical law problem include recursive formula, function induction mode, Yang Hui triangle and function interpolation. Based on the analysis, the teaching enlightenment is proposed: pay attention to the development of number sense and improve the abstraction ability; Pay attention to mathematical connections and cultivate divergent thinking; Highlight mathematical reasoning and cultivate reasoning ability.

***Keywords**: numerical regularity; functional inductive mode; function interpolation;*

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I. Ask questions

Numerical law problems refer to a class of problems that give a column of numbers, observe and summarize the rules, and guess the next number. In recent years, such questions have often appeared in various examinations such as primary and secondary schools, civil service examinations, and teacher recruitment examinations. When solving the problem of number regularity, students need to combine the characteristics of the number between numbers and their own mathematical cognition level, and go through the thinking process of observation, guessing, experiment, reasoning, argumentation and reflection, so that it is possible to discover the law, get a reasonable answer, and finally reach the realm of "don't let the clouds cover your eyes, and get rid of the prosperity to know the true face". In mathematics teaching, this kind of problem can let students experience the methods of mathematical discovery, accumulate experience in mathematical activities, and improve the flexibility, profundity, extensiveness and agility of students' mathematical thinking quality, which is an important material for cultivating students' number sense, intuitive imagination, logical reasoning and other core literacy of mathematics, especially in cultivating students' innovative thinking. With the deepening of the reform of mathematics curriculum in China, the new curriculum clearly requires that evaluation should not only pay attention to the learning results, but also pay attention to the learning process of students, in addition to evaluating the mastery of students' knowledge and skills, but also to examine the thinking process, depth and breadth of thinking of students through the design of tools such as open questions. [1] Therefore, numerical problems will still play an important role in students' mathematics learning. The following expounds the methods and ideas of solving this kind of problem through the analysis of a numerical law problem, in order to provide a reference for the teaching of this kind of law problem.

II. Analyze The Basic Strategies Of Numerical Law Problems

Problem presentation [2]: Find the pattern first, and then fill in the appropriate number in parentheses. 0,1,4,15,56, ( ). This question comes from a question in the Olympiad of Primary 4 Mathematics.

Recursive formula

Analysis: If the relationship between adjacent two or more items of a sequence can be expressed by a formula, then this formula is called the recursive formula of the sequence. Students observe the quantitative relationship between numbers, and can find that 1 and 4 have a 4-fold relationship, and 4 and 15 have a nearly 4-fold relationship with 15 and 56, as shown in Table 1. Therefore, from Table 1, we can get $4 = 4 \times 1 - 0; 15 = 4 \times 4 - 1; 56 = 4 \times 15 - 4$, and the law is summarized as $a_n = 4a_{n-1} - a_{n-2}$ $(n > 2, n \in N, a_1 = 0, a_2 = 1)$.

Table 1

Serial number	1	2	3	4	5	6
predicted values	0	1	4	15	56	209
Decomposition representation	0	1	4×1-0	4×4-1	4×15-4	4×56-15

The step of "review and reflection" is an intrinsic part of the process of mathematical problem solving, which is a kind of active thinking activity and exploration behavior, and is the source of exploration, discovery and creation, which is conducive to cultivating students' ability to raise problems and innovative spirit, is conducive to students discovering new laws and expanding, extending and promoting, is conducive to improving students' mathematical literacy and improving their true understanding of mathematics, and is a key link that cannot be ignored in mathematical problem solving, learning and research [3], when the problem has been solved, And looking back at the problems that have been solved can be found and created. So what angles can this question reflect on? What is the general formula for this set of arrays? Can it be promoted? Can you find an answer from another perspective?

Combined with the above analysis, the general formula is obtained: $a_n = \left(\frac{\sqrt{3}}{3} - \frac{1}{2}\right)\left(2 + \sqrt{3}\right)^n + \left(-\frac{\sqrt{3}}{3} - \frac{1}{2}\right)\left(2 - \sqrt{3}\right)^n, (n \geq 1, n \in N)$.

Analysis: Knowing the recursive formula $a_n = 4a_{n-1} - a_{n-2}(n > 2, n \in N, a_1 = 0, a_2 = 1)$, the general formula of the a_n is obtained by using the eigenroot equation method. Let $\frac{a_n}{a_{n-1}} = \frac{a_{n-1}}{a_{n-2}} = x(n \geq 3)$, the characteristic equation of the recursive series is: $x^2 = 4x - 1$.Solution: $x_1 = 2 + \sqrt{3}; x_2 = 2 - \sqrt{3}$, then $a_n = c_1 {x_1}^n + c_2 {x_2}^n$, and$a_1 = c_1 {x_1}^1 + c_2 {x_2}^1 = 0$, $a_2 = c_1 {x_1}^2 + c_2 {x_2}^2 = 1$, the solution is: $c_1 = \frac{\sqrt{3}}{3} - \frac{1}{2}; c_2 = -\frac{\sqrt{3}}{3} - \frac{1}{2}$, then $a_n = \left(\frac{\sqrt{3}}{3} - \frac{1}{2}\right)\left(2 + \sqrt{3}\right)^n + \left(-\frac{\sqrt{3}}{3} - \frac{1}{2}\right)\left(2 - \sqrt{3}\right)^n, (n \geq 1, n \in N)$. Continuing to generalize the coefficients in the recursive formula a_n, the form is as follows: $a_1 = a$, $a_2 = b$, $a_n = pa_{n-1} - qa_{n-2}(n \geq 3)$ (p, q are constants), and the solution method is the same as above.

Functional induction mode

Analysis: The function induction model refers to the way of thinking that the number of special cases under investigation is arranged sequentially to form a sequence, and these quantities are regarded as the function values of a function f(n) about a positive integer n, and then the function relational is found by separating the constant from the independent variable n or the function known by association [4].

Let $a_1 = 0$, $a_2 = 1$, $a_3 = 4$, $a_4 = 15$, $a_5 = 56$, the question is $a_6 = ?$, according to the inductive mode of the function, let $a_n = (n-1)b_n$, $n \geq 2$, $n \in N$, so, $b_2 = 1$, $b_3 = 2$, $b_4 = 5$, $b_5 = 14$. And because $b_3 - b_2 = 1$, $b_4 - b_3 = 3$, $b_5 - b_4 = 9$ note that 1,3,9, the latter term is 3 times that of the previous term, guess $b_6 - b_5 = 27$, then $b_6 = 41$, $a_6 = 5 \times 41 = 205$. According to this law, the general formula for the b_n of the accumulation method is $b_n = \frac{3^{n-2}+1}{2}(\mathrm{n} \geq 2, \mathrm{n} \in \mathrm{N})$, that is, $a_n = (n-1)\left(\frac{3^{n-2}+1}{2}\right), (n \geq 2, n \in N, a_1 = 0)$.

The key to this method is to first express the number as the product of a continuous natural number and a certain number, that is, let $a_n = (n-1)b_n$, $n \geq 2$, $n \in N$, and then observe the regular characteristics of the remaining numbers.

Yang Hui Triangle

Analysis: Yang Hui triangle appeared in the book "Detailed Explanation of Nine Chapters of Algorithm" written by Yang Hui, a mathematician in the Southern Song Dynasty of China, in 1261, as a mathematical treasure of excellent traditional Chinese culture, it appears in textbooks of various grades. [5] The Yanghui triangle has many important properties, which enable students to appreciate the symmetrical beauty, simple beauty, and singular beauty of mathematics in the process of learning. When analyzing the four numbers 1, 4, 15, and 56, you can analyze them from two perspectives.

First, when looking at "1,4,15,56" from the number sense, it is associated with 4=C_4^1, 15=C_6^2, and 56 can be expressed as C_8^3, then this column of numbers is expressed by a combination number: C_2^0, C_4^1, C_6^2, C_8^3, and the fifth number is speculated to be C_{10}^4, that is, 210, so the general formula, that is, $a_n = c_{2n-2}^{n-2} = \frac{(2n-2)!}{n!(\mathrm{n}-2)!}, (n \geq 2, a_1 = 0)$.

Second, from the Yang Hui triangle (Figure 1), the Yang Hui triangle appears in China's primary, junior high and high school textbooks, and it is found that it is a vertical column in Figure 1 (as shown in the box number in Figure 1), and 210 can be obtained. With the deepening of learning, the key to solving such problems is to strengthen the ability to represent the same object in different forms or increase the sensitivity to numbers. In the past, the Yanghui Triangle was mostly observed from the horizontal perspective and less from the longitudinal angle, so familiarizing with the Yanghui Triangle is one of the important ways to solve this kind of problem.

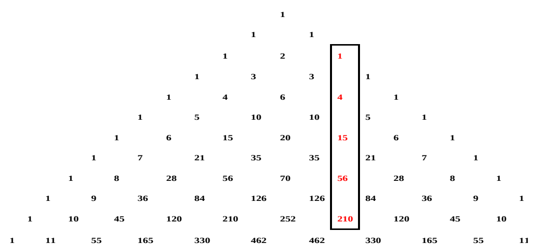

Fig.1 Yang Hui Triangle (partial)

Function interpolation method

Analysis: The above three perspectives depend on the sensitivity to numbers, so is there a more general way? First of all, list this row of numbers into a table, as shown in Table 2 below, as 5 points, the coordinates are (1,0), (2,1), (3,4), (4,15), (5,56),Knowing the value of the function $y = p(\mathrm{x})$at $n+1$ different points of the function $f(x_i) = y_i \quad (i = 1,2,\dots,n)$(i=1,2,...,n), we want to construct a polynomial function y=f(x) such that $f(x_i) = y_i \quad (i = 1,2,\dots,n)$ This method is called polynomial interpolation.

Let $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$, and substitute the above 5 sets of values to get $a_0 = 11$, $a_1 = -\frac{73}{3}$, $a_2 = \frac{55}{3}$, $a_3 = \frac{-17}{3}$, $a_4 = \frac{2}{3}$, then $f(x) = \frac{2x^4-17x^3+55x^2-73x+33}{3}$, when $x = 6$, $f(6) = 165$.

Table 2

x						
$f(x)$				5	6	

Perspective 3 and perspective 1 are both applications of function ideas, and perspective 3 is more general and versatile than perspective 1. This method is more suitable for junior high school and above, the key of the polynomial function interpolation method is to guide students to think about the problem generally, in the junior high school stage through the non-collinear 3 points can find the analytical formula of the unary quadratic function, in which the undetermined coefficient method is used, let $f(x) = a_0 + a_1x + a_2x^2$, then inspire students to know the coordinates of 5 points, using the same method, then let the analytical formula be $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. Finally, the answer can be obtained by solving the five-element one-dimensional equation.

III. Pedagogical Implications

Pay attention to the development of number sense and improve abstraction ability

The above four methods are the basic methods to deal with the problem of numerical law, which provides a useful reference for solving such problems in the future. In addition, there are some methods that need to guide students to pay attention to accumulation, such as the general term calculation method of the higher-order equal difference series, the calculation method of the general term of the generalized Fibonacci series, and the method of finding the parent function of the number series, which are all effective strategies to deal with the problem of number laws. Most of these methods rely on the observation, operation and induction of data to make conjectures, which shows that this mental process requires students to have a good sense of numbers. Therefore, teachers should pay attention to the development of students' number sense, so as to further develop abstract ability, and at the same time, teachers should lose no time in guiding students to master and apply these basic methods, so as to provide strong support for students to solve such problems.

Pay attention to mathematical connections and cultivate divergent thinking

Through the analysis of a numerical law problem from multiple perspectives, it can be found that many seemingly unrelated problems in mathematics will be linked together with the in-depth exploration of the problem from different perspectives. Among the four perspectives, perspectives 1, 2 and 3 are to discover the potential unique quantitative model law by observing and combining the quantitative characteristics of special cases, which has the characteristics of complex thinking and low calculation difficulty. The processing method of perspective 4 is to approximate the answer to the problem through polynomial function interpolation, which has the characteristics of concise thinking and difficult calculation. The general formula behind the four answers is different, each way of thinking has its merits, and each different answer has its rationality. Therefore, in the teaching process, it is necessary to guide students to think and solve problems from different perspectives and different knowledge points within mathematics.

Highlight mathematical reasoning and cultivate reasoning ability

Numerical law questions often appear in the form of single choice or fill-in-the-blank questions, giving people the impression of objective questions, but because some questions may have different solution ideas and different answers, the answers may not be unique, which then arouses everyone's doubts about this kind of questions as objective questions. In view of the fact that there may be a situation where the answer to the numerical law question is not unique, some scholars suggest that this kind of question should be regarded as a subjective question, as long as the candidate writes what he thinks is the law and can justify himself, he should be given points according to the situation[6]. In addition, it has been suggested that such a title could be replaced with "how many terms may be"[7]. It can be seen that in mathematics teaching, when encountering such problems, teachers should allow students to have different answers, instead of blindly taking the uniqueness of the results as the criterion for judging, when students give their own answers, they should give students a certain amount of time to fully explain the thinking process, as long as students can give a reasonable explanation for the methods they propose, they should give students affirmation, and develop logical reasoning literacy in the process of speaking and reasoning.

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